CALCULUS 110

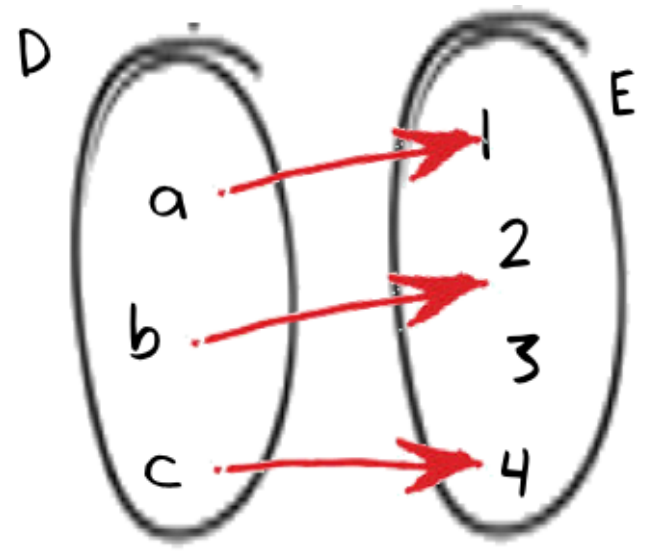
1.1 Four ways to represent a function

Verbally-numerically- visually (graph) - algebraically

Four ways to represent a function

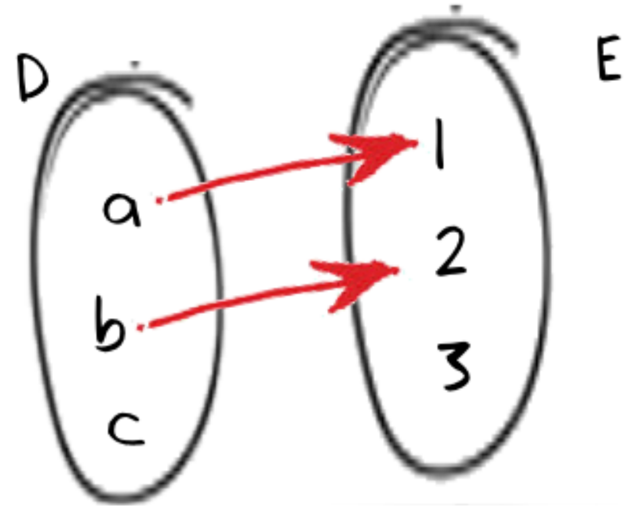
Function:

A Function f is a rule that assigns to each element × € D exactly one element, called f(x) € E
We usually consider functions for which the sets D and E are sets of real numbers. f: R→R



Domain is the value of x

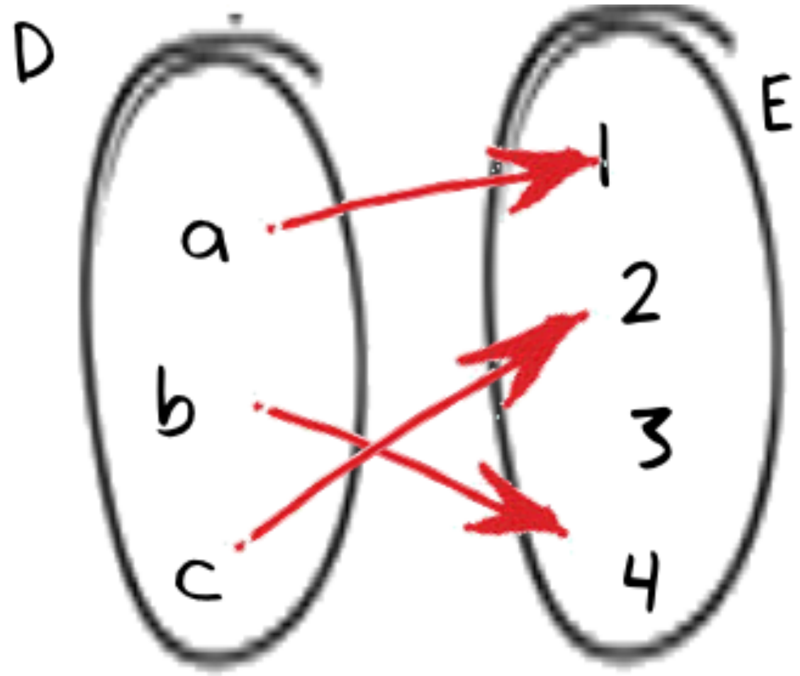
Range is the set of all possible values

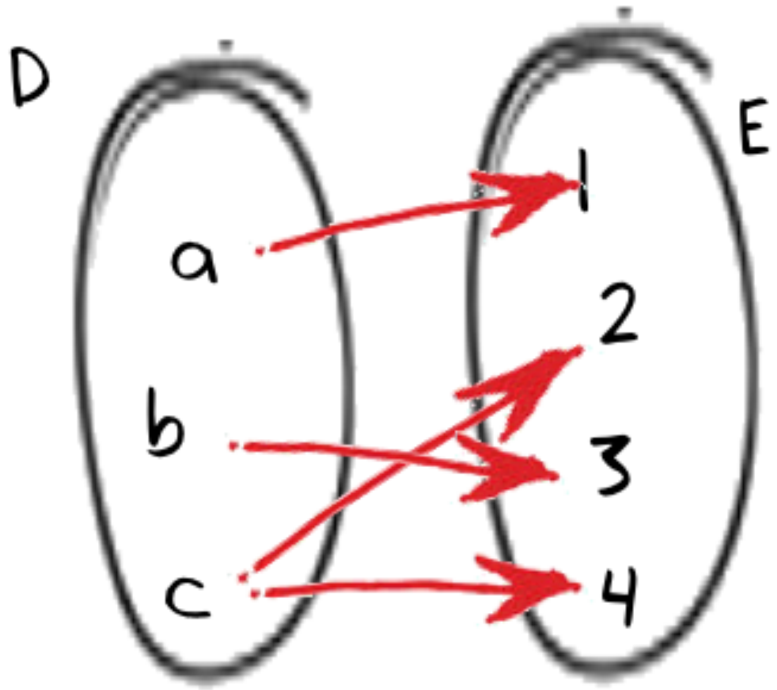


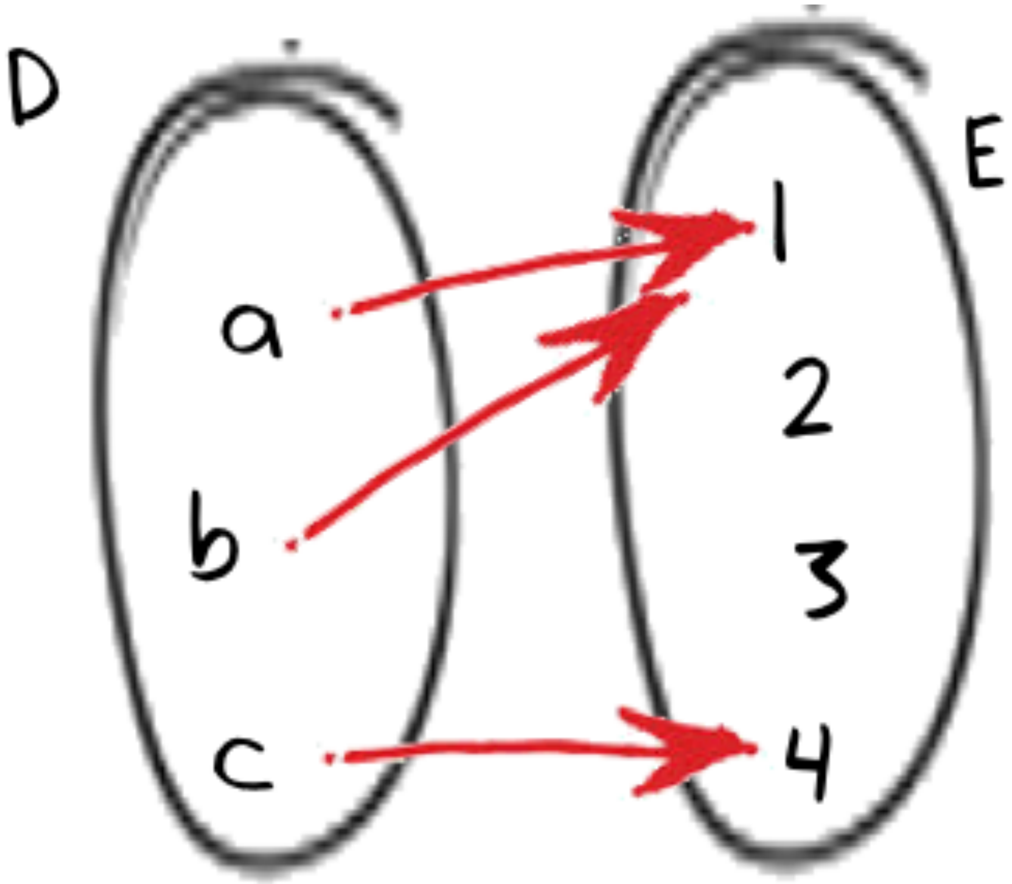
remark :

1- The set D is called the Domain of this function .
2- The sat E is called the Codomain of the function.
3-The set of all images y= f(x)∈E s.t CX∈D is called the range.
4-Range ⊆ Codomain → الـ Range مجموعة جزئية من الـ codomain
5- x € D is called independent varible (متغير مستقل).
F(x)=y Range of the function is called dependent Variable(متغير تابع)

four ways to represent a function







Domain = _____

Range = _____

Codomain = _____

The graph of a function f gives us a useful picture of the behavior or “life history” of a function. Since the $y-coordinate$ of any point (x, y) on the graph is $y = f(x)$, we can read the value of $f(x)$ from the graph as being the height of the graph above the point x (see Figure 4). The graph of f also allows us to picture the domain of f on the $x-axis$ and its range on the $y-axis$ as in Figure 5.

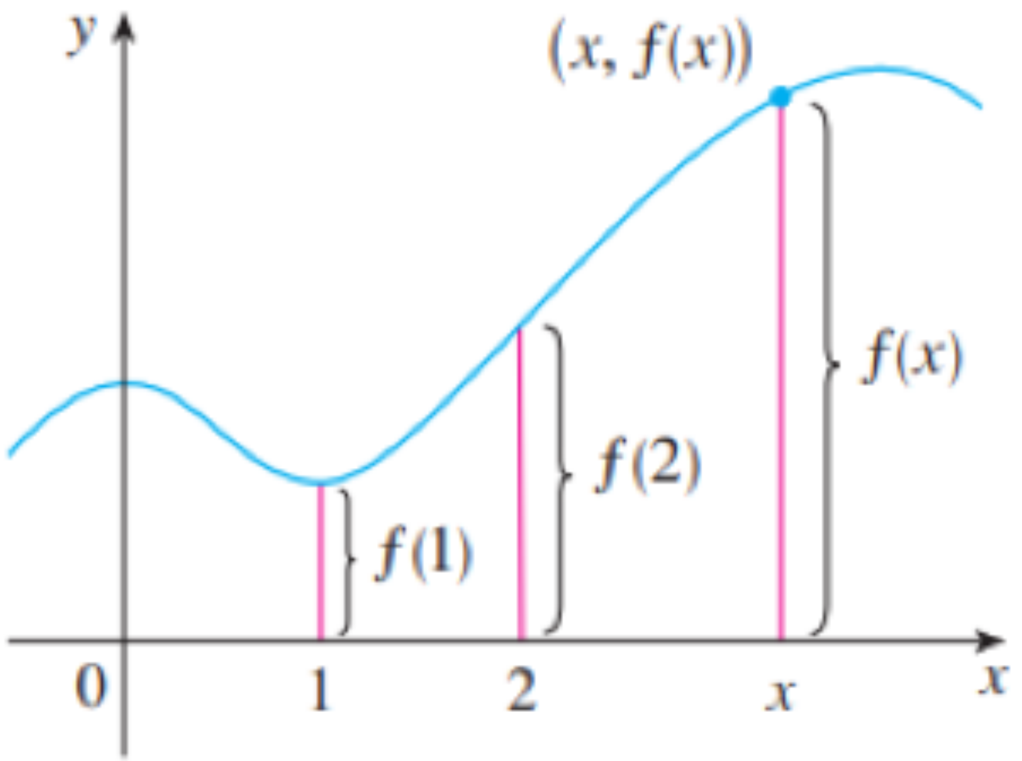


FIGURE 4

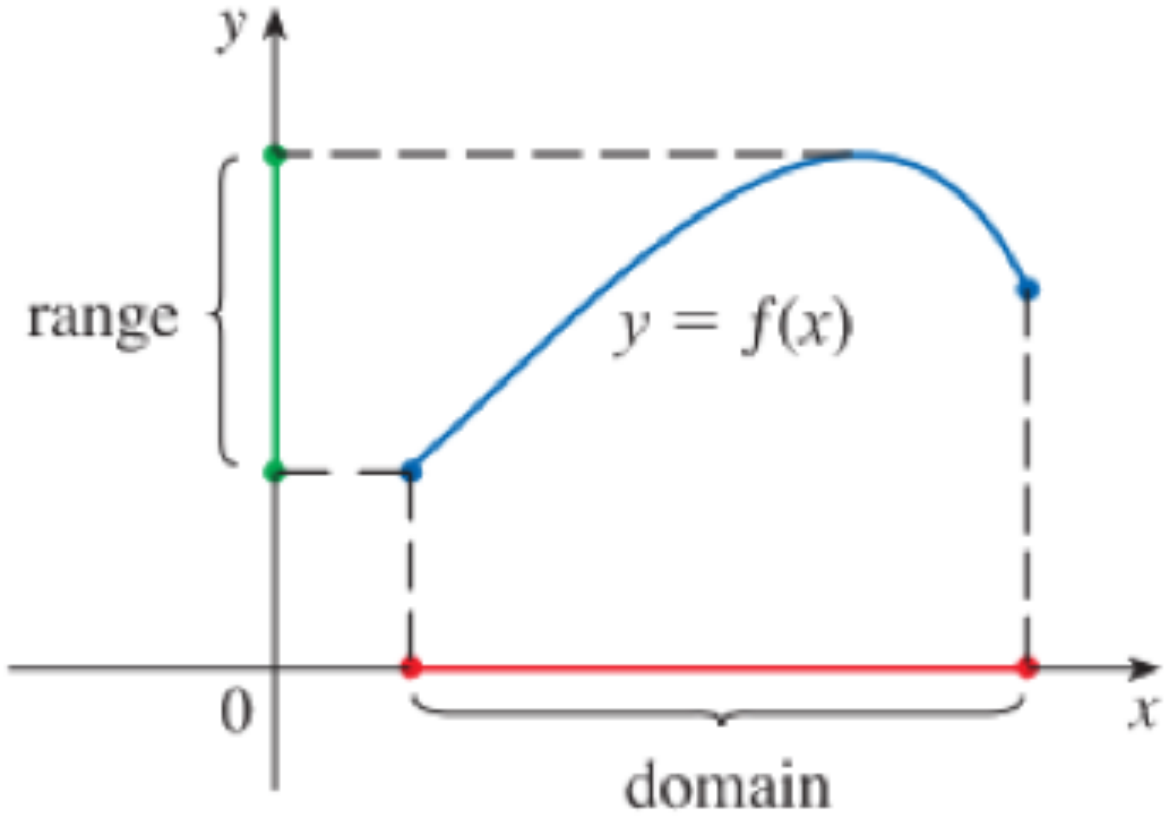


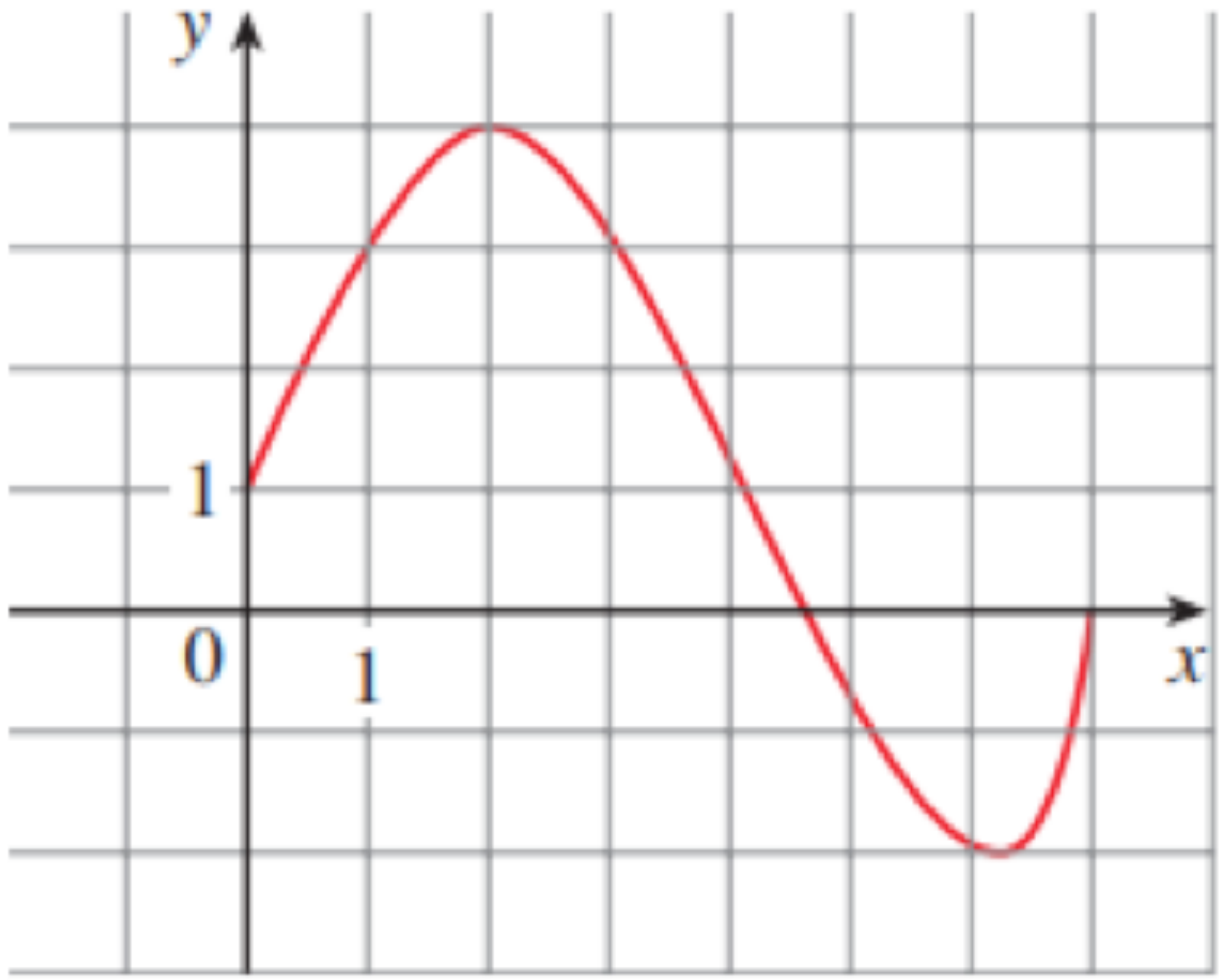
FIGURE 5

Example 1

The graph of a function f is shown in the following Figure.

(a) Find the values of f(1) and f(3).

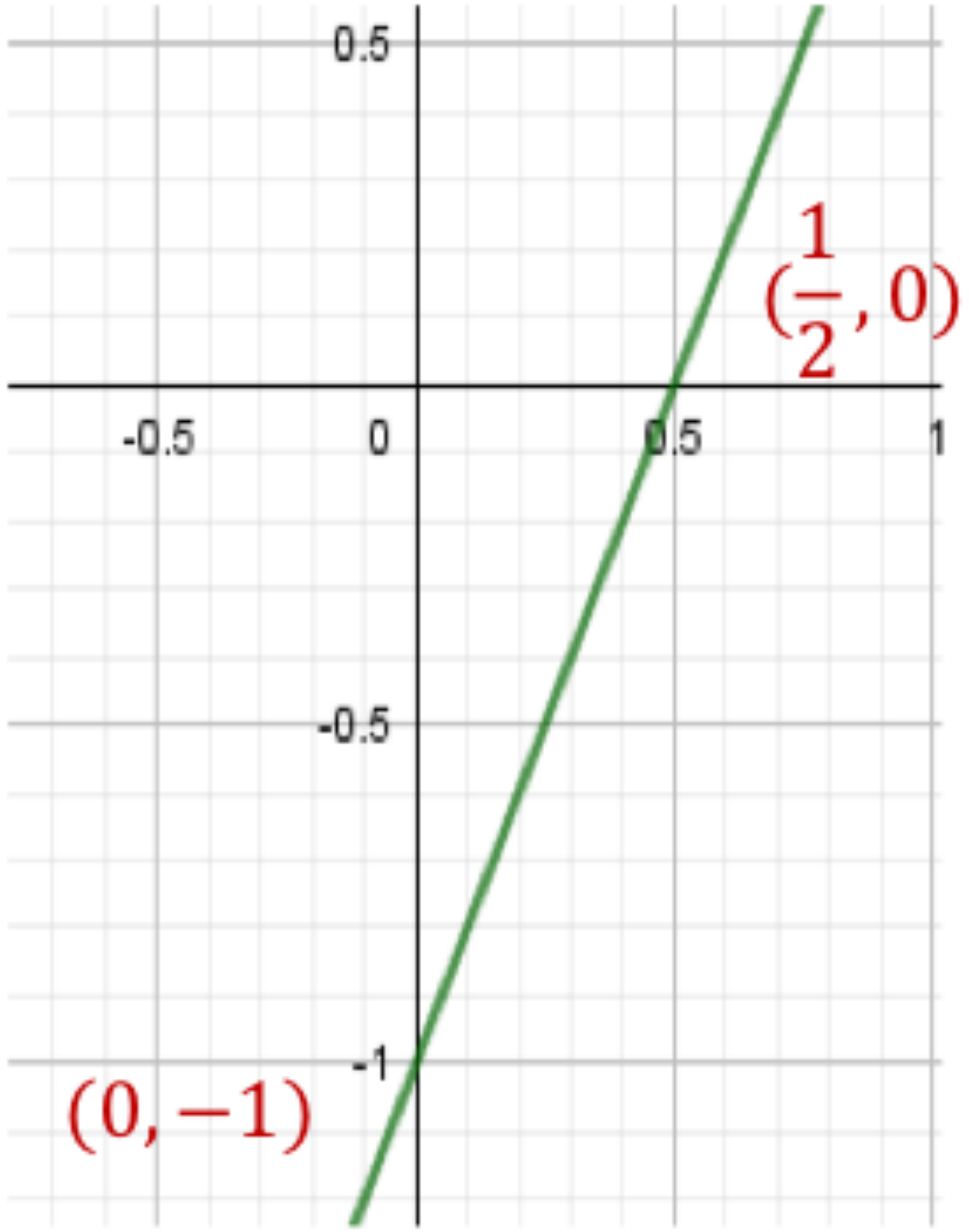
(b) What are the domain and range of f?



Example 2

Sketch the graph and find the domain and range of each function.

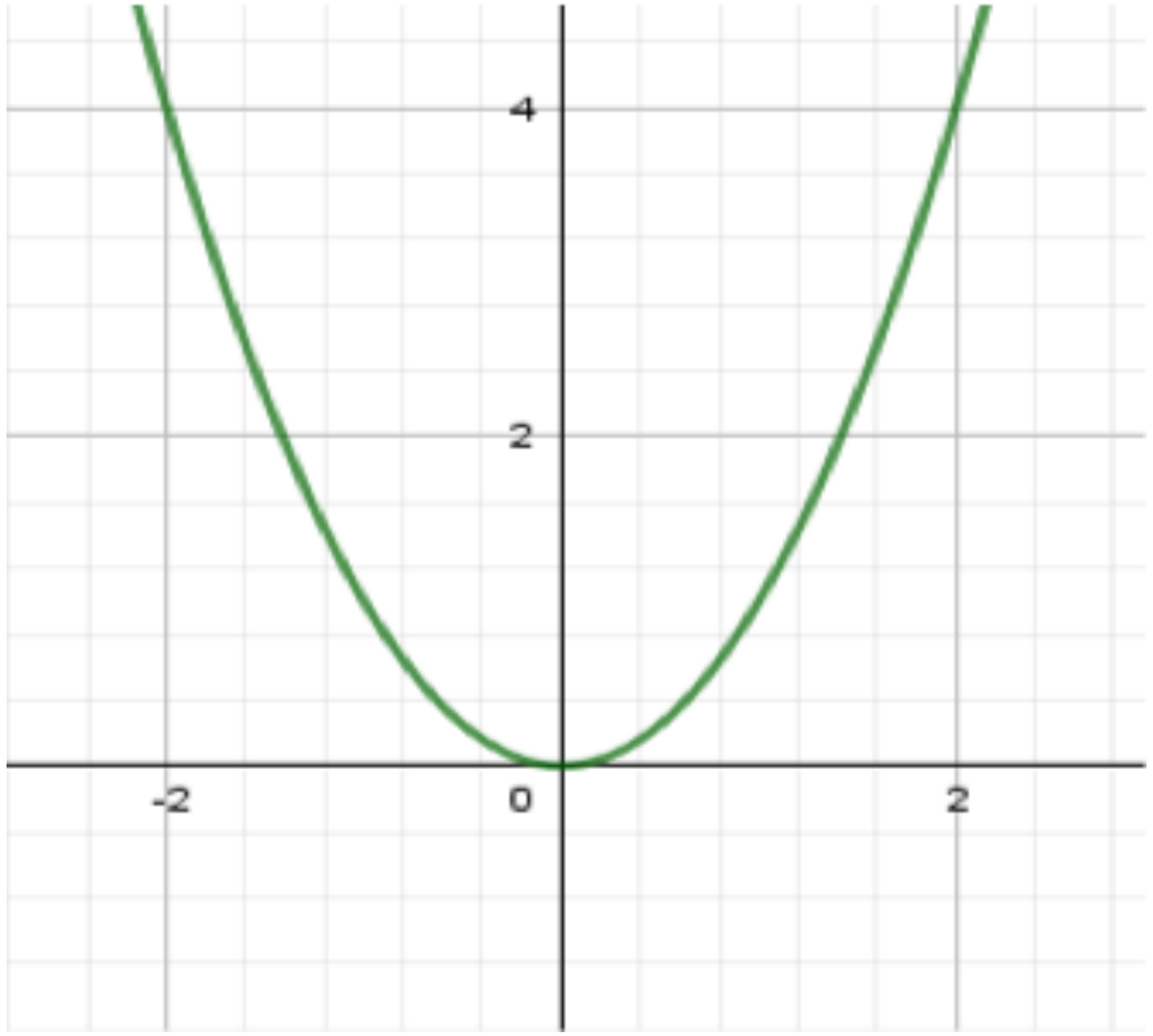
$(a)\ f(x) = 2x - 1$



Example 2

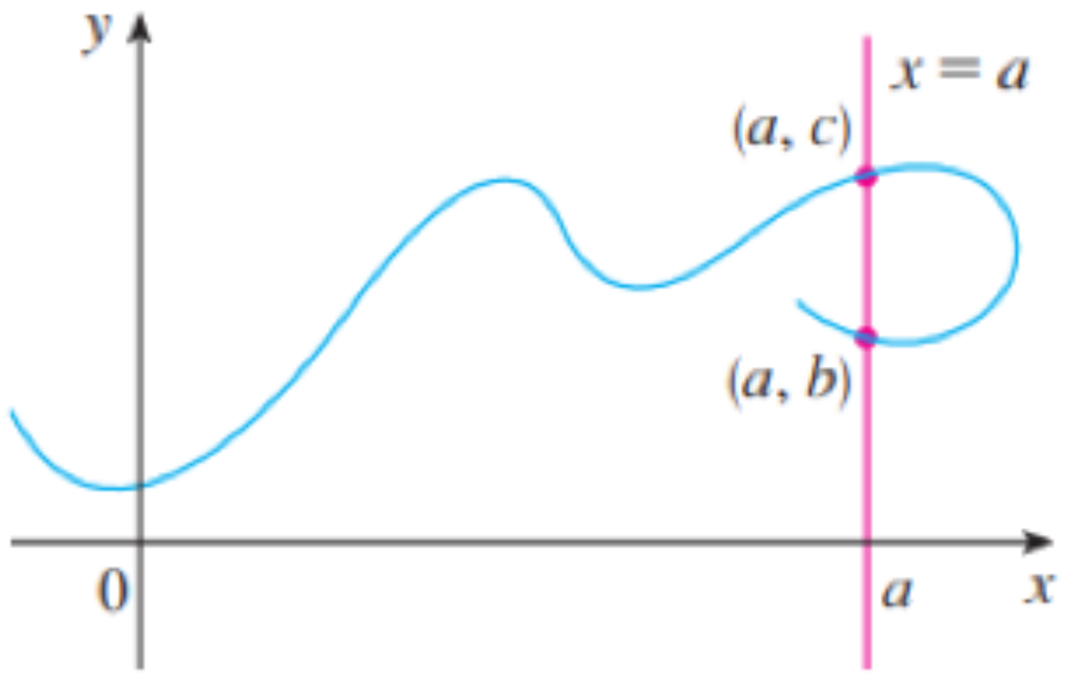
Sketch the graph and find the domain and range of each function.

$(b) \quad g(x) = x^2$

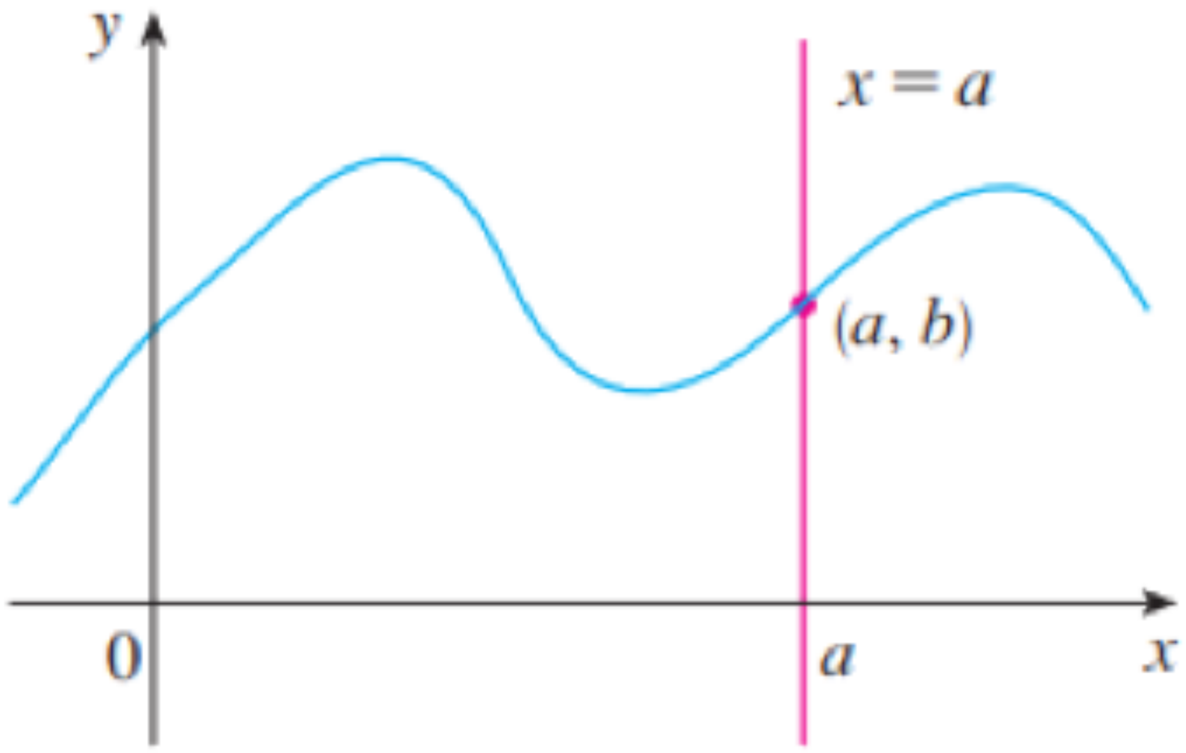


The vertical line Test

A curve in the xy plane is the graph of a function of x if no vertical line intersects the curve more than once.



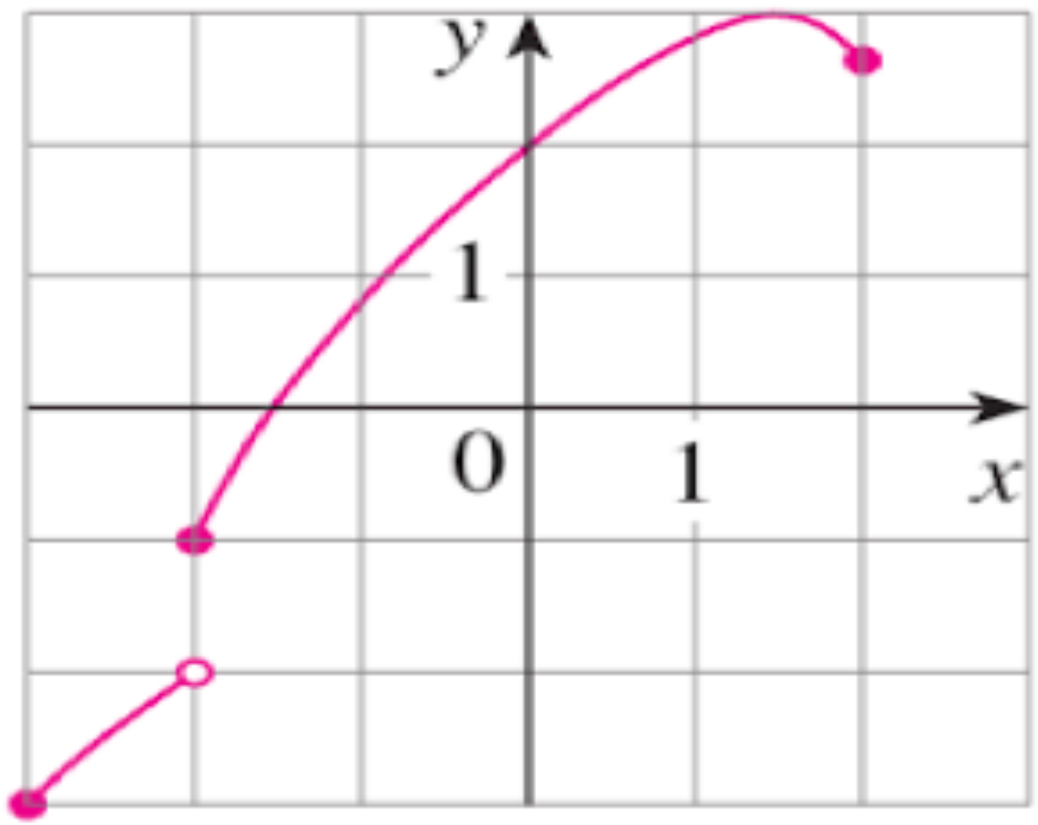
(b) This curve doesn't represent a function.



(a) This curve represents a function.

Exercises 17

Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.



domain=

range=

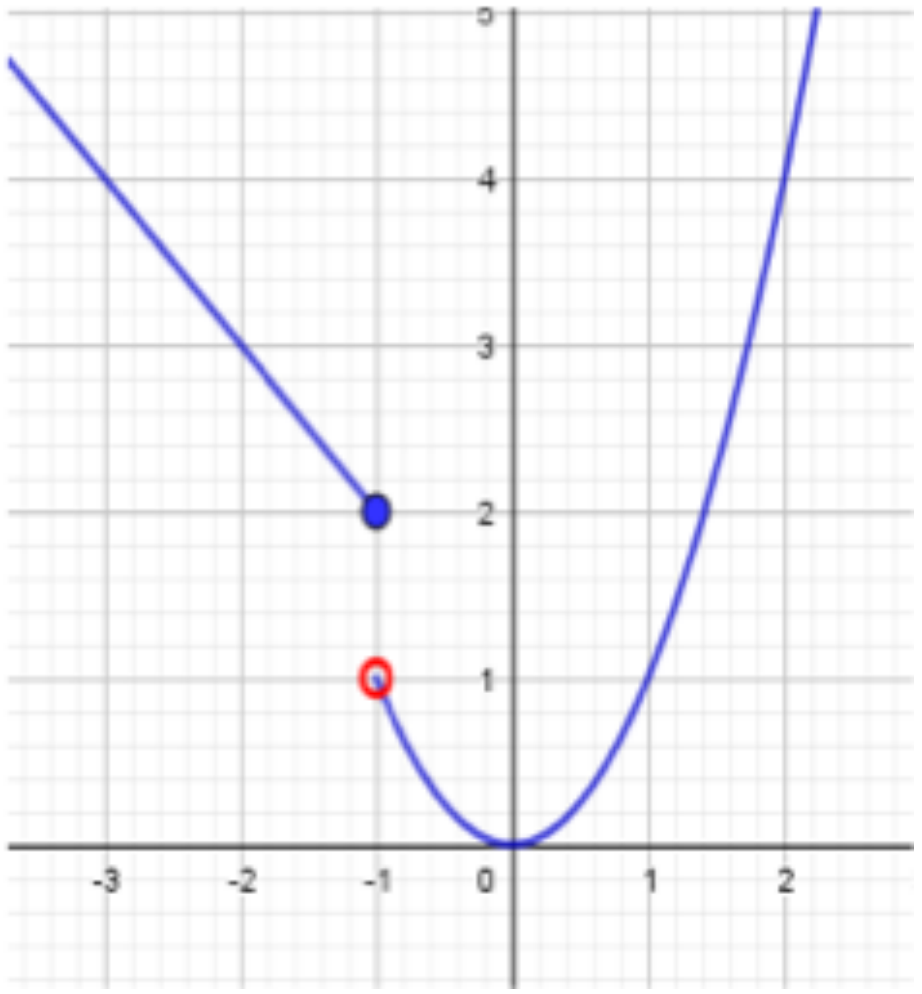
Piecewise function

is a function that is defined by different formulas in different parts of their domains.

Example 7

$$f(x) = \begin{cases} 1 - x, & x \le -1 \\ x^2, & x > -1 \end{cases}$$

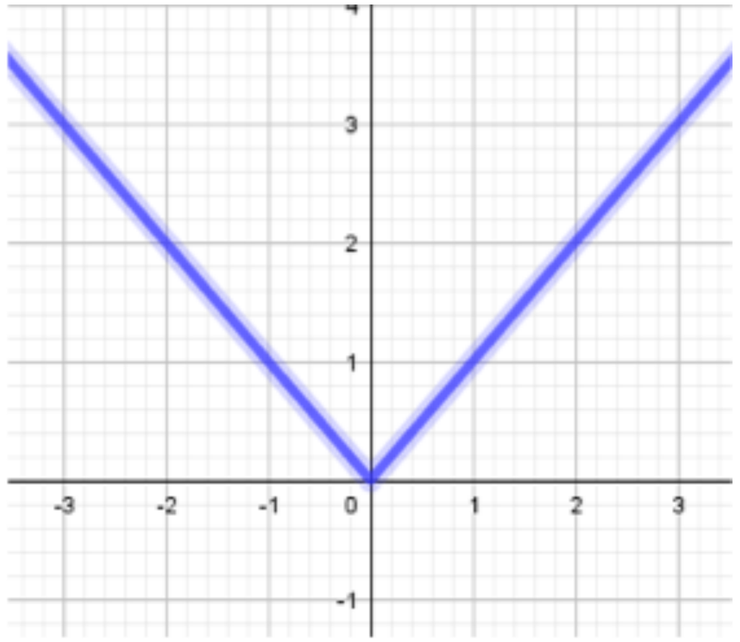
Evaluate f(-2), f(-1) and f (0) and sketch the graph.



Example 8

Sketch the graph of

$$f(x) = |x| = \begin{cases} x & if \;\; x \geq 0 \\ -x & if \;\; x < 0 \end{cases}$$



Domain

Range

Properties of Absolute Values

Suppose a > 0 is any real number .Then

(1) $|x| = |-x|$

(4) $|x| = a \Leftrightarrow x = \pm a$

(2) $|x| \leq a \Leftrightarrow -a \leq x \leq a$. $\{x \in [-a, a]\}$

(3) $|x| \geq a \Leftrightarrow x \geq a$ or $x \leq -a$. $\{x \in (-\infty, -a] \cup [a, \infty)\}$

$\sqrt{a^2} = a$ if $a \geq 0.$ $\sqrt{(-a)^2} = -a$ if $a < 0.$

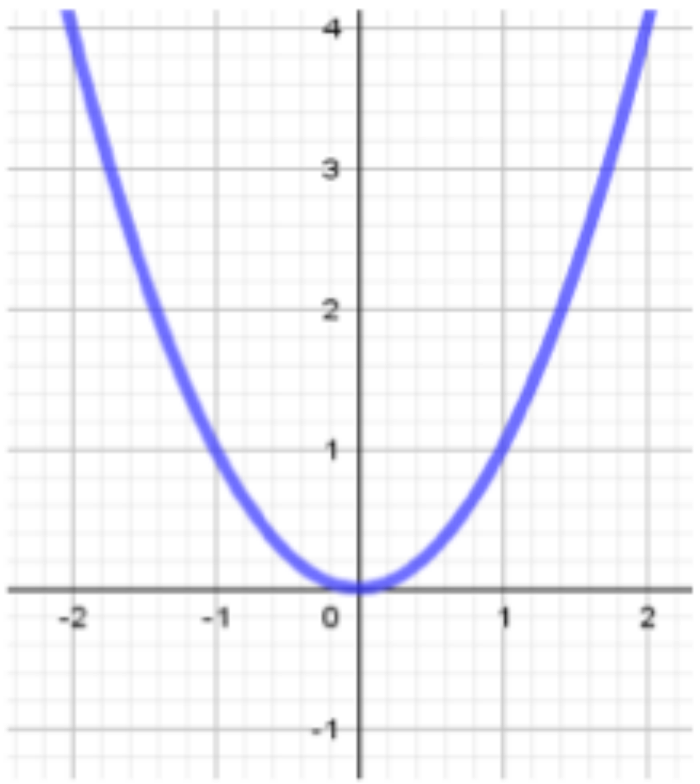
$$\sqrt{a^2} = |a|$$

Symmetry

Even function

if $f(-x) = f(x)\ \forall\ x \in D$, then f is called an even function.

The graph is symmetric with **respect to the _ y _ axis .**

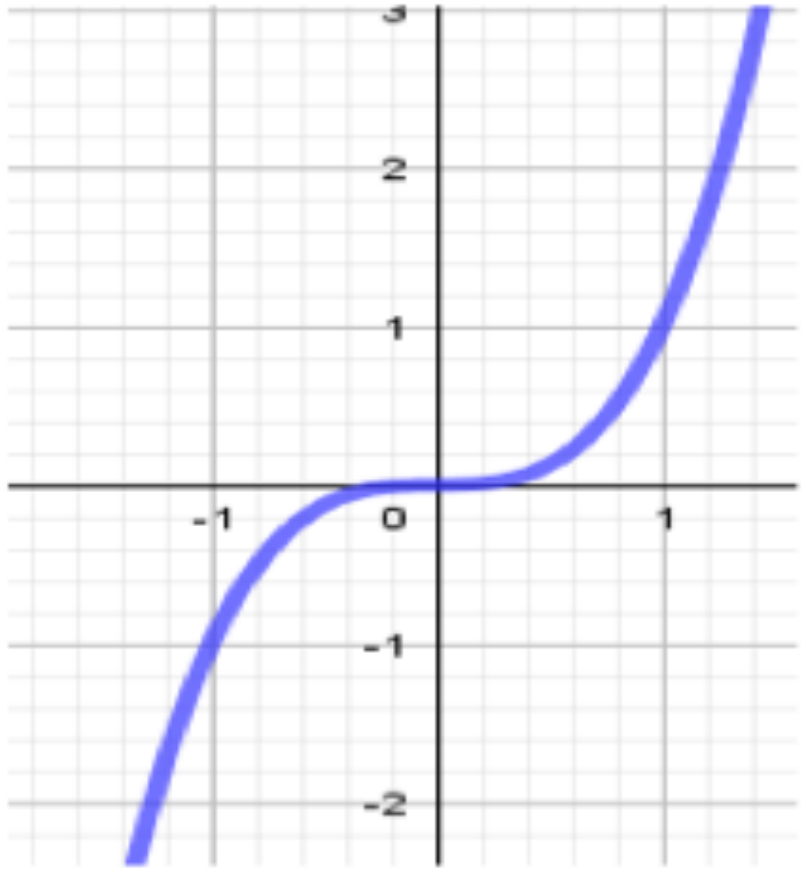


$f(x) = x^2$ is even

Odd function

If $f(-x) = -f(x)\ \forall\ x \in D$, then f is called an odd function.

The graph is **symmetric with respect to the _origin_ .**



$f(x) = x^3$ is odd

Notes:

1-The re are functions that are neither even nor odd

2- There is no function that is even and odd at the same time. ⇒ $f(x) = 0$

3- for f(x)= x^n , $n \in Z^+$

if n is even

⇒ f(x) is even function

if n is odd

⇒ f(x) is odd function

All Trigonometric Functions are odd functions except $cosx$ and $secx$ are even functions.

Adding (subtracting):

The sum of two even functions is even
The sum of two odd functions is odd
The sum of an even and odd function is neither even nor odd (unless one function is zero).

even ± even → even
odd ± odd → odd
odd ± even → neither

Multiplying (division):

The product of two even functions is an even function.
The product of two odd functions is an even function.
The product of an even function and an odd function is an odd function.

even × even → even
odd × odd → even
odd × even → odd

Example 11

Determine whether each of the following functions is even, odd or neither even nor odd.

(a) $f(x) = x^5 + x.$

(b) $f(x) = 1 - x^4$

(c) $f(x) = 2x - x^2$

(a) $f(x) = c$ is _ _ _ _ _

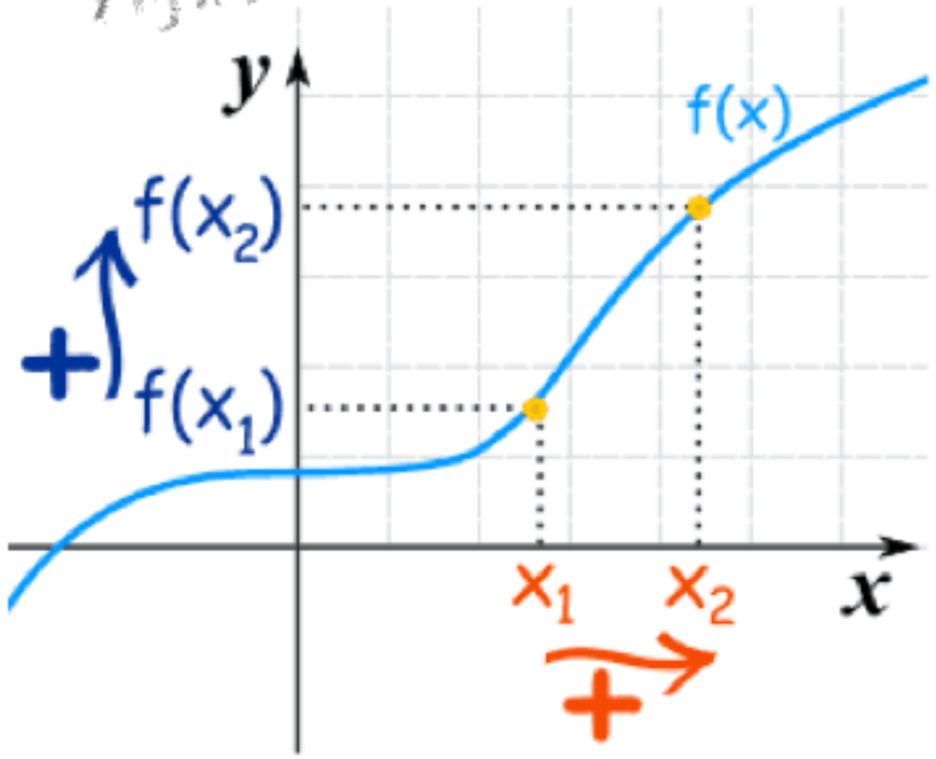
(b) $f(x) = |x|$ is _ _ _ _ _

(c) $f(x) = x^n$ is $\begin{cases} \text{\_ \_ \_} & \text{if } n \text{ is even} \\ \text{\_ \_ \_ \_ \_,} & \text{if } n \text{ is odd} \end{cases}$

Increasing and Decreasing

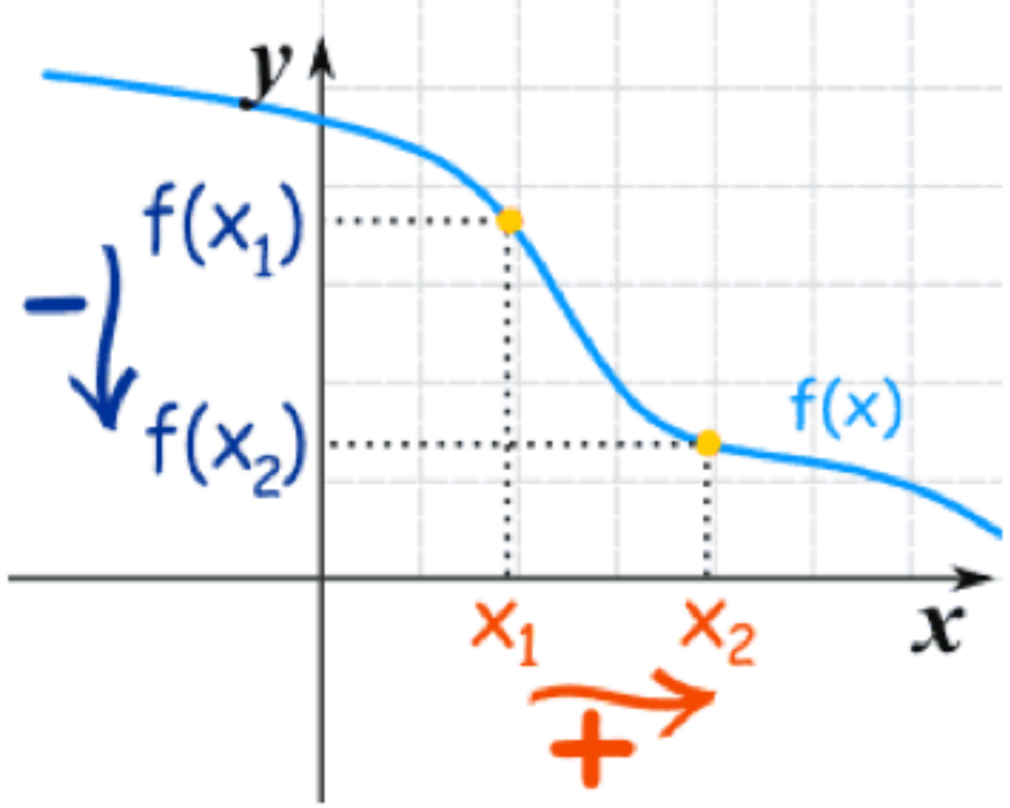
A function f is called **increasing** on an interval I

if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.



A function f is called **decreasing** on an interval I

$if\ f(x_1) > f(x_2)\ ,\ x_1 < x_2$ in I .

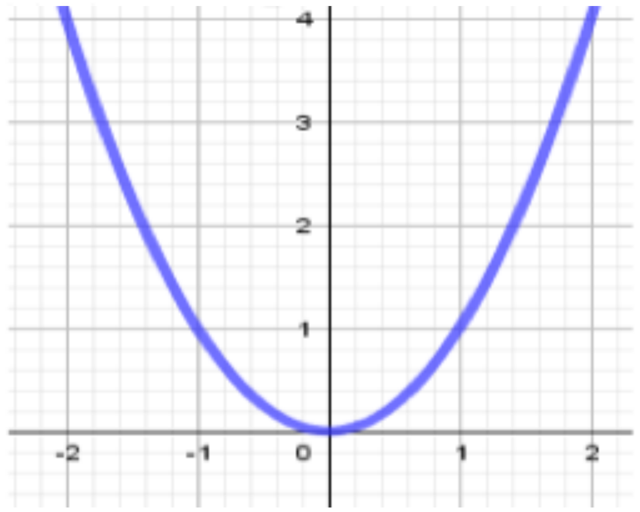


Example :

$f(x) = x^2$

$f(x)$ is decreasing $(-\infty, 0]$.

$f(x)$ is increasing $[0,\infty)$.

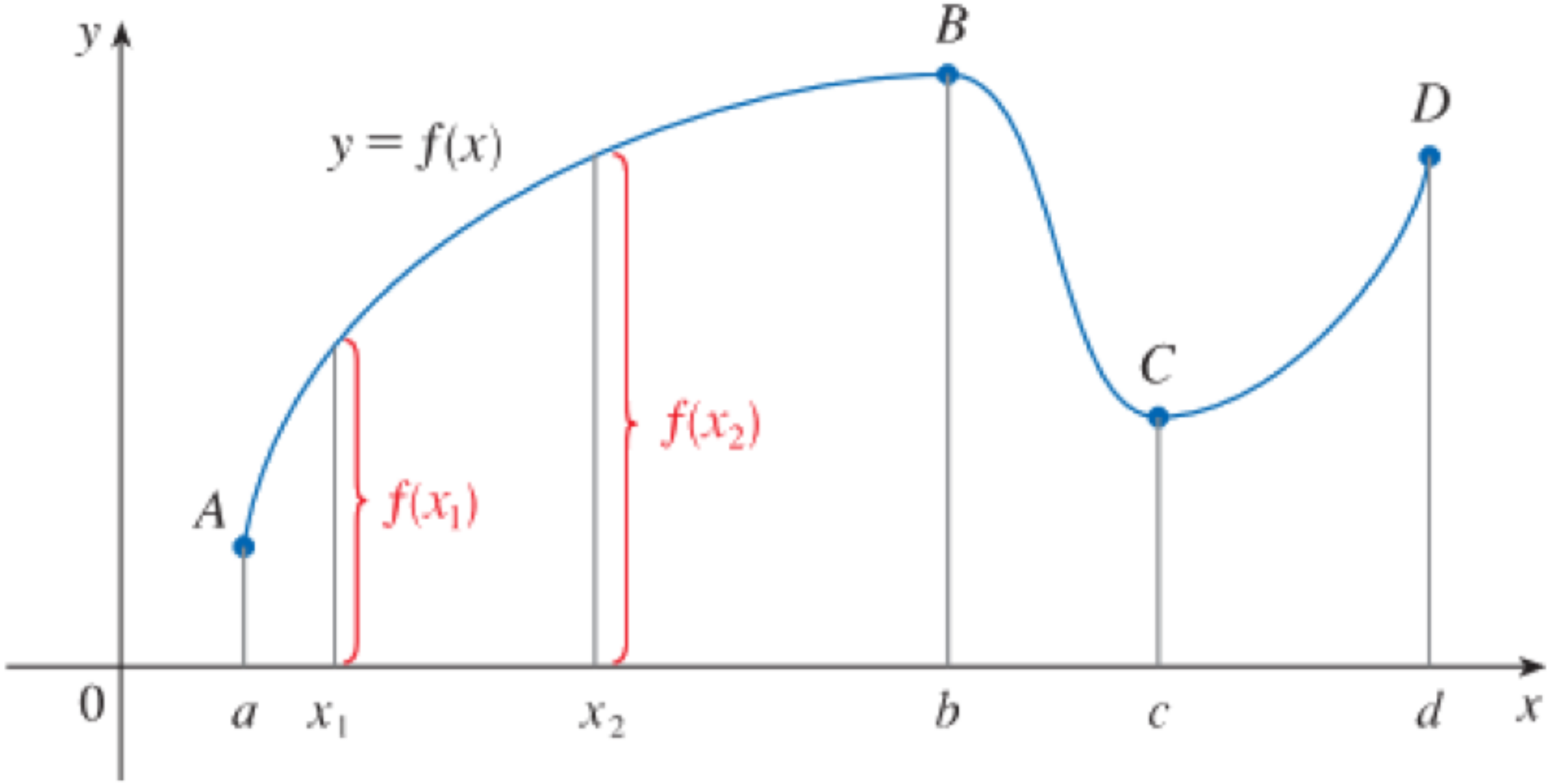


A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on I if

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$



Example

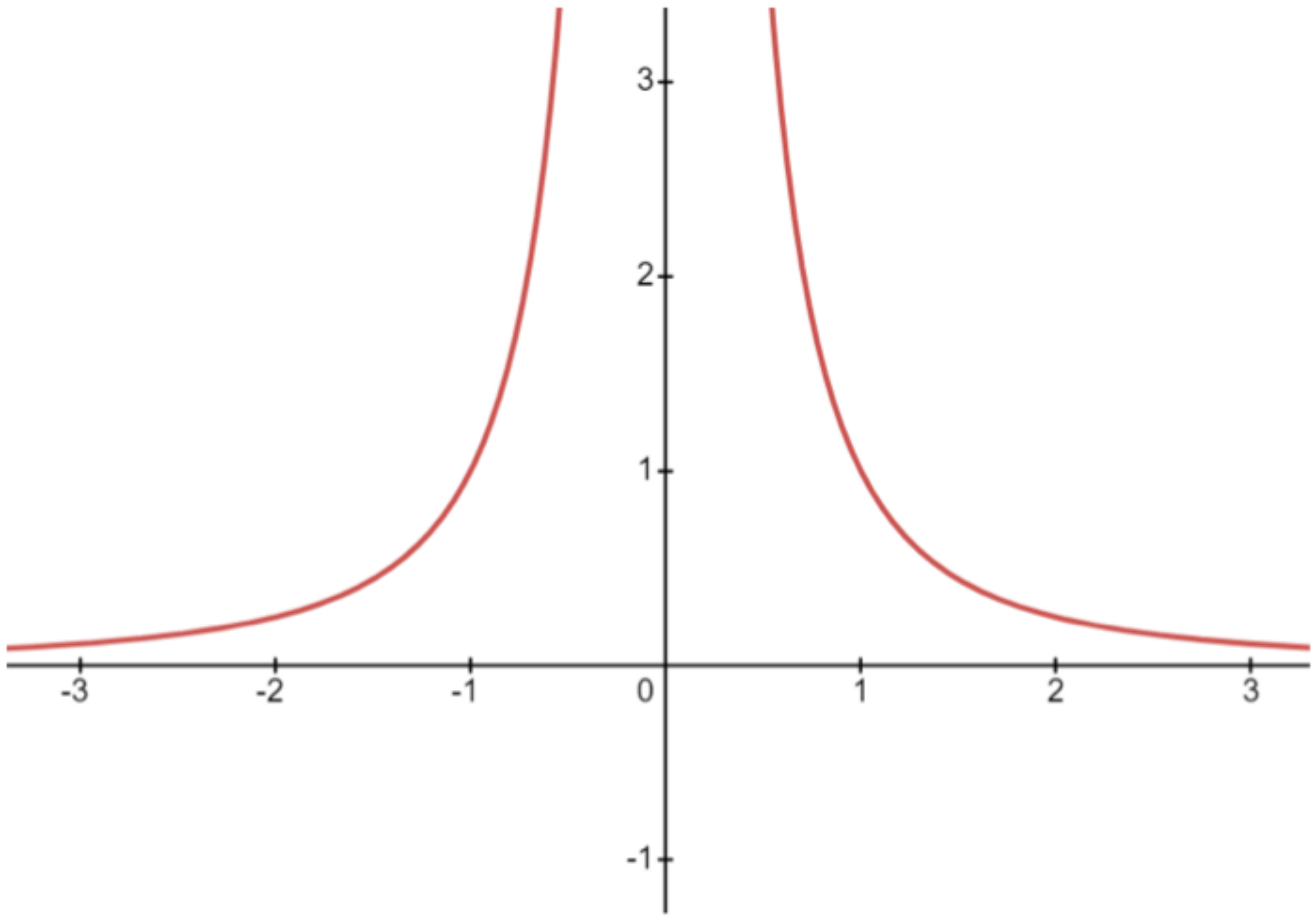
The function whose graph is given is

(a) Increasing on $(-\infty, 0)$.

(b) Increasing on $(0, \infty)$.

(c) decreasing on $(-\infty, 0)$.

(d) decreasing on $\mathbb{R}$.



Homework

8. The graph of a function g is given.

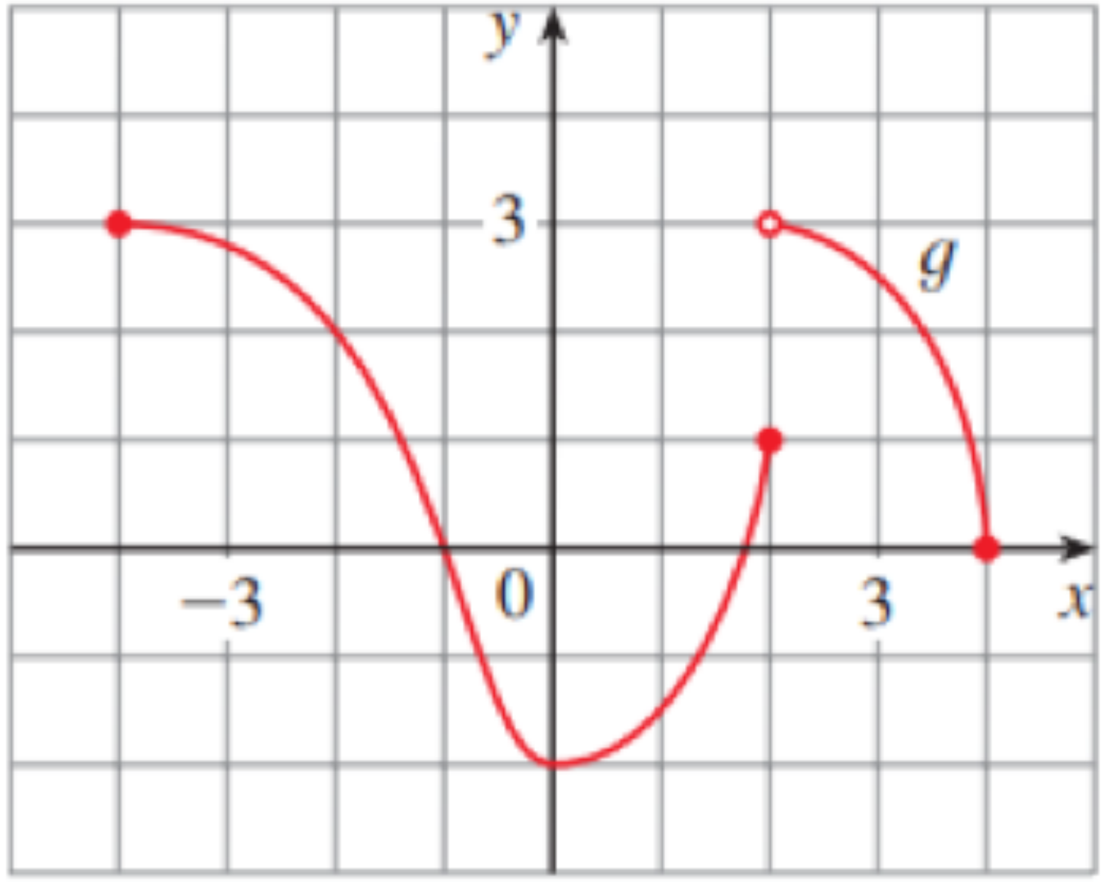
(a) State the values of $g(-2)$, $g(0)$, $g(2)$, and $g(3)$.

(b) For what value(s) of x is $g(x) = 3$?

(c) For what value(s) of x is $g(x) \leqslant 3$?

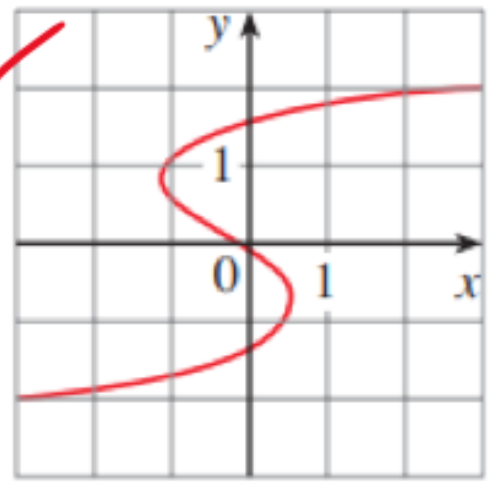
(d) State the domain and range of g.

(e) On what interval(s) is g increasing?



15–18 Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.

15.



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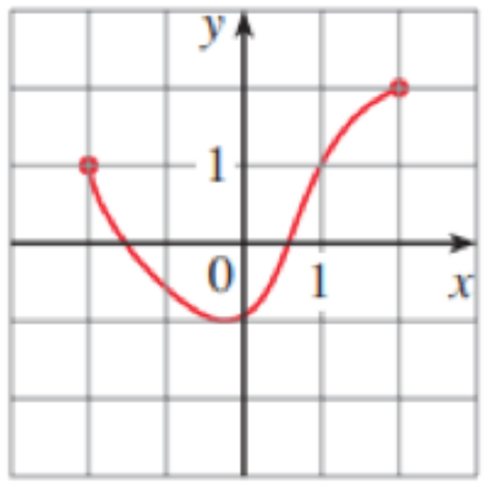

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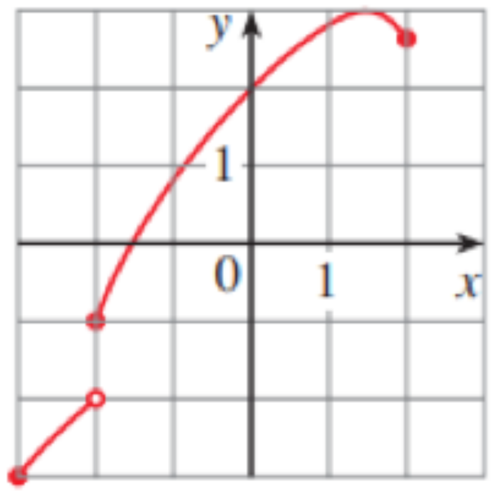

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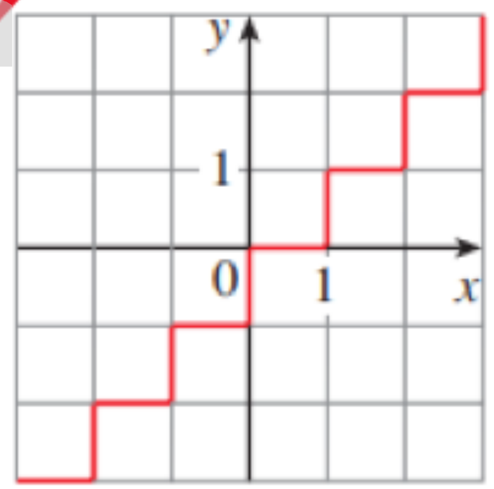


EXAMPLE 6 Find the domain of each function.

(a) $f(x) = \sqrt{x + 2}$

(b) $g(x) = \dfrac{1}{x^2 - x}$

Find the domain of the function.

39. $f(x) = \dfrac{x + 4}{x^2 - 9}$

Exercise 41

$f(t) = \sqrt[3]{2t - 1}$

49–52 Evaluate $f(-3)$, $f(0)$, and $f(2)$ for the piecewise defined function. Then sketch the graph of the function.

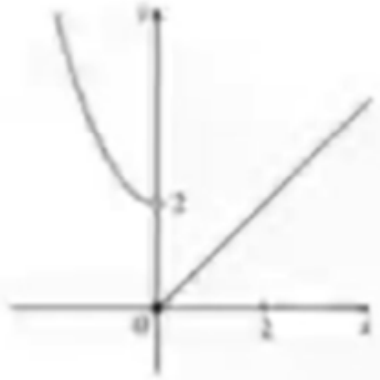
49. $f(x) = \begin{cases} x^2 + 2 & \text{if } x < 0 \\ x & \text{if } x \geqslant 0 \end{cases}$

50. $f(x) = \begin{cases} 5 & \text{if } x < 2 \\ \frac{1}{2}x - 3 & \text{if } x \geqslant 2 \end{cases}$

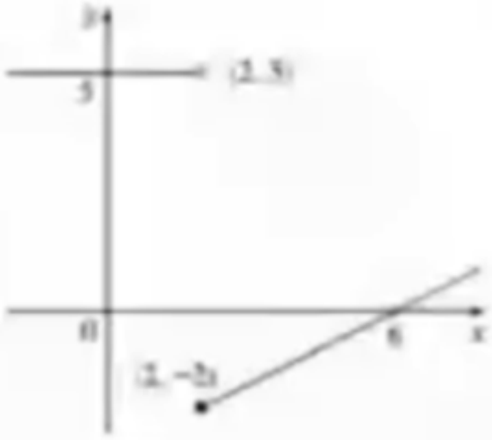
51. $f(x) = \begin{cases} x + 1 & \text{if } x \leqslant -1 \\ x^2 & \text{if } x > -1 \end{cases}$

52. $f(x) = \begin{cases} -1 & \text{if } x \leqslant 1 \\ 7 - 2x & \text{if } x > 1 \end{cases}$

49. $f(x) = \begin{cases} x^2 + 2 & \text{if } x < 0 \\ x & \text{if } x \geqslant 0 \end{cases}$



50. $f(x) = \begin{cases} 5 & \text{if } x < 2 \\ \frac{1}{2}x - 3 & \text{if } x \geqslant 2 \end{cases}$



51. $f(x) = \begin{cases} x + 1 & \text{if } x \leqslant -1 \\ x^2 & \text{if } x > -1 \end{cases}$

52. $f(x) = \begin{cases} -1 & \text{if } x \leqslant 1 \\ 7 - 2x & \text{if } x > 1 \end{cases}$

Sketch the graph of the function.

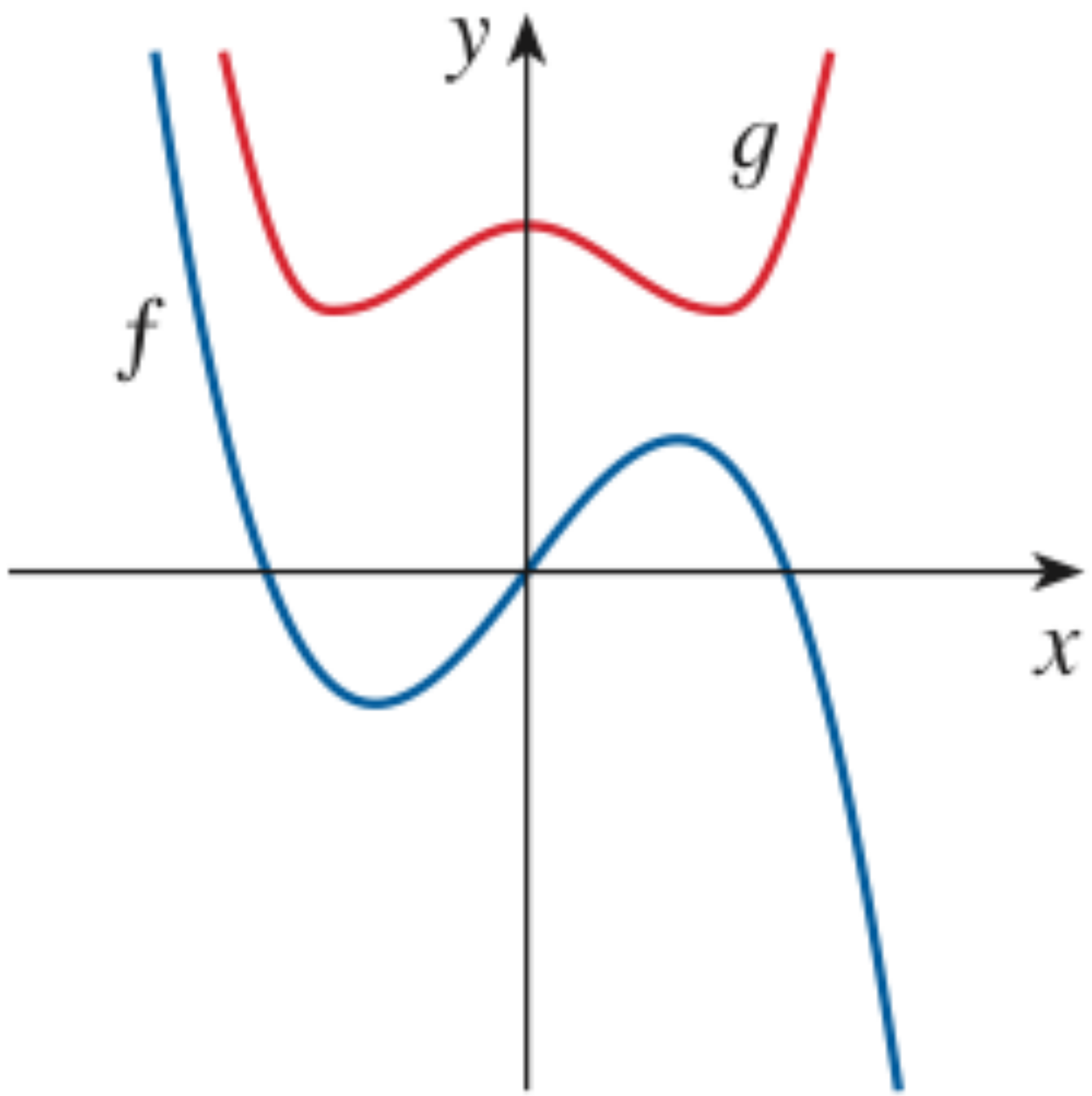
54. $f(x) = |x + 2|$

57. $f(x) = \begin{cases} |x| & \text{if } |x| \leqslant 1 \\ 1 & \text{if } |x| > 1 \end{cases}$

56. $f(x) = \dfrac{|x|}{x}$

77–78 Graphs of f and g are shown. Decide whether each function is even, odd, or neither. Explain your reasoning.

77.



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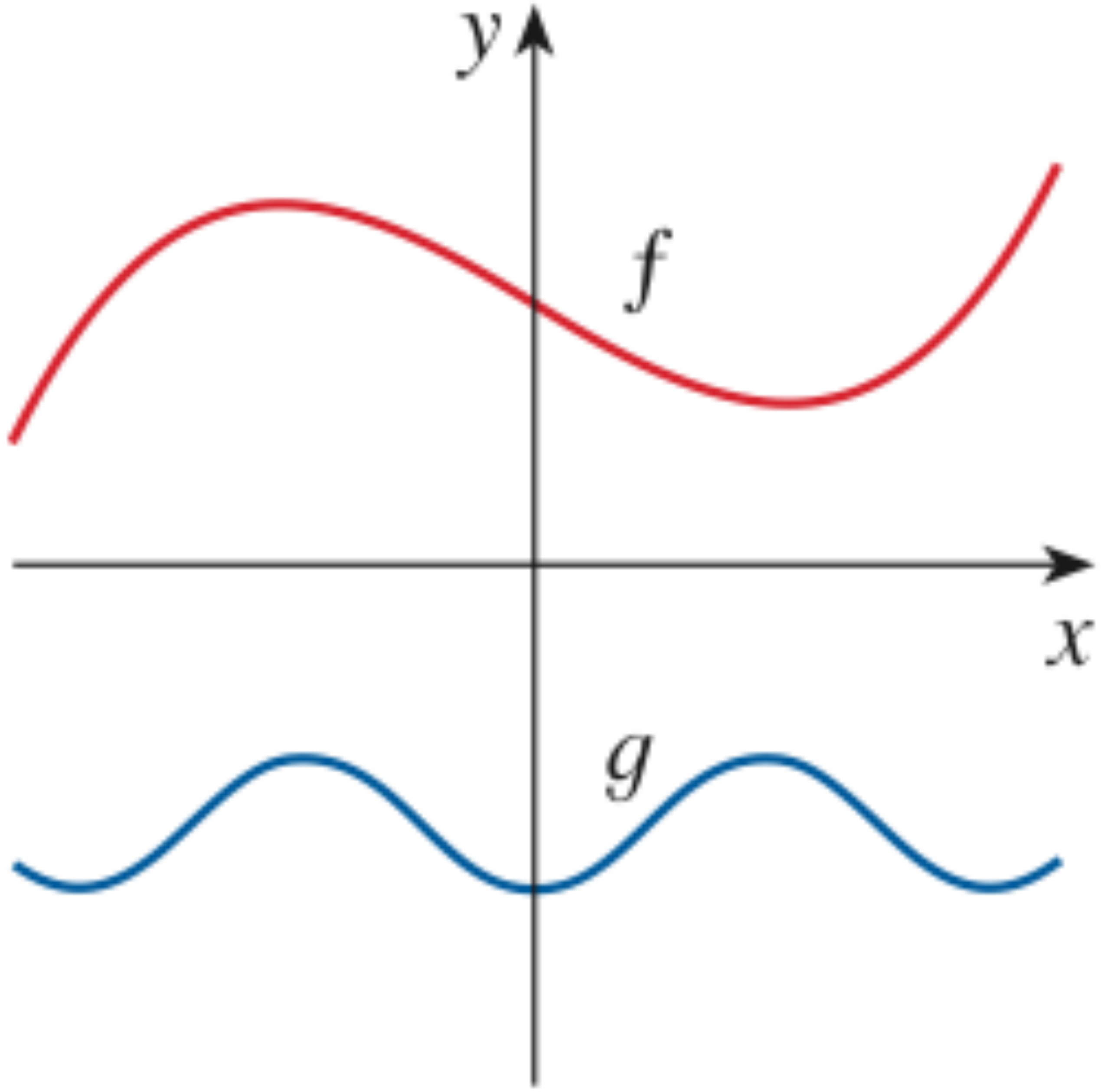

81–86 Determine whether f is even, odd, or neither. You may wish to use a graphing calculator or computer to check your answer visually.

81. $f(x) = \dfrac{x}{x^2 + 1}$

82. $f(x) = \dfrac{x^2}{x^4 + 1}$

83. $f(x) = \dfrac{x}{x + 1}$

84. $f(x) = x|x|$

85. $f(x) = 1 + 3x^2 - x^4$

86. $f(x) = 1 + 3x^3 - x^5$

Done✅

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